St. Petersburg State University
Petrozavodsk State University

STRUCTURE AND EVOLUTION OF STELLAR SYSTEMS

Proceedings of the International Conference

Edited by T.A. Agekian, A.A. Mülläri, V.V. Orlov

Petrozavodsk, Karelia, Russia
13–17 August 1995

St. Petersburg
1997

тического диска большая толщина малометалличной подсистемы звезд поддерживается падением на диск бедной металлами межгалактической материи.

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FINITE MIXTURES OF STELLAR SYSTEMS: MOMENTS AND CUMULANTS AS STATISTICS OF KINEMATIC PARAMETERS

R. Cubarsi[1], S. Alcobe[2]

[1]*Universitat Politècnica de Catalunya, Barcelona, Spain*
[2]*Universitat de Barcelona, Barcelona, Spain*

We consider the total distribution function as the obtained by superposition of two normal distribution functions associated with the corresponding populations – ($'$) or ($''$) for the first or second population, according to $f = f' + f''$. We shall deduce the parameters of the total velocity distribution starting from those of the partial distributions. For the stellar density N, by defining $n' = N'/N$, $n'' = N''/N$, we get $1 = n' + n''$. And, for the mean velocity $\mathbf{v}$, we obtain the following equation: $\mathbf{v} = n'\mathbf{v}' + n''\mathbf{v}''$. Furthermore, the total central moments $\mathbf{M}_n$ can be computed (*Cubarsi, 1992*) from the partial ones by using the centroid differential velocity $\mathbf{w} = \mathbf{v}' - \mathbf{v}''$. It is possible to express these relationships by introducing the total cumulants and the following new variables:

$$\mathbf{D} = \sqrt{n'n''}\,\mathbf{w};\ q = \sqrt{n'/n''} - \sqrt{n''/n'}$$

(we can appoint the populations so that $n' \geq n''$; then q is non-negative) and also we define the following second-rank tensors:

$$\mathbf{a}_2 = n'\mathbf{K}'_2 + n''\mathbf{K}''_2$$
$$\mathbf{C}_2 = (\mathbf{K}'_2 - \mathbf{K}''_2)/\sqrt{q^2+4} - q(\mathbf{D})^2 \ .$$

With all the latter definitions the total cumulants of the mixture satisfy

$$\begin{aligned}\mathbf{K}_2 &= \mathbf{a}_2 + (\mathbf{D})^2 \ , \\ \mathbf{K}_3 &= Sym(\mathbf{C}_2 \otimes \mathbf{D}) + 2q(\mathbf{D})^3 \ , \\ \mathbf{K}_4 &= \tfrac{1}{2}Sym(\mathbf{C}_2 \otimes \mathbf{C}_2) - 2(q^2+1)(\mathbf{D})^4 \ .\end{aligned}$$

We are interested in the general case where the centroid differential velocity (and vector $\mathbf{D}$) is not null. Hence let us assume $D_3 \neq 0$ (we can assume that this component corresponds to $max_i\,|D_i|$), and let us define a normalized vector $\mathbf{d} = \mathbf{D}/D_3$ with the direction containing both subcentroids. Since every normal population distribution is symmetric with respect to its centroid, the total velocity distribution will be symmetric in whatever direction within a perpendicular plane to $\mathbf{d}$. Thus, in order to simplify the problem, it is convenient to work with some transformed vector $\mathbf{W}$ of the peculiar velocity $\mathbf{u}$, which contains two vector components within this plane. For this reason we take these directions as $(d_3, 0, -d_1)^t$ and $(0, d_3, -d_2)^t$, which are orthogonal to the vector $\mathbf{d}$. Two of the vector components of the new vector will be the projections of $\mathbf{u}$ to these directions, and the third component will not be modified. Thus, the new vector $\mathbf{W}$ is the obtained from the following isomorphic transformation of vector $\mathbf{u}$:

$$\mathbf{W} = \mathbf{H}_2 \cdot \mathbf{u}; \ \mathbf{H}_2 = \begin{pmatrix} d_3 & 0 & -d_1 \\ 0 & d_3 & -d_2 \\ 0 & 0 & 1 \end{pmatrix}.$$

Now we can calculate the cumulants –in particular the third and fourth– of $\mathbf{W}$, in function of the corresponding ones of $\mathbf{u}$:

$$o_{111} = -\kappa_{333}d_1^3 + 3\kappa_{133}d_1^2 d_3 - 3\kappa_{113}d_1 d_3^2 + \kappa_{111}d_3^3,$$

$$o_{222} = -\kappa_{333}d_2^3 + 3\kappa_{332}d_3 d_2^2 - 3\kappa_{322}d_3^2 d_2 + \kappa_{222}d_3^3,$$

$$o_{112} = -\kappa_{333}d_1^2 d_2 + \kappa_{332}d_1^2 d_3 + 2\kappa_{133}d_1 d_3 d_2 - \\ -2\kappa_{132}d_1 d_3^2 - \kappa_{113}d_3^2 d_2 + \kappa_{112}d_3^3,$$

$$o_{122} = -\kappa_{333}d_1 d_2^2 + \kappa_{133}d_3 d_2^2 + 2\kappa_{332}d_1 d_3 d_2 - \\ -2\kappa_{132}d_3^2 d_2 - \kappa_{322}d_1 d_3^2 + \kappa_{122}d_3^3,$$

$$p_{22} = \kappa_{333}d_2^2 - 2\kappa_{332}d_3 d_2 + \kappa_{322}d_3^2,$$

$$p_{12} = \kappa_{333}d_1 d_3 d_2 - \kappa_{332}d_1 d_3 - \kappa_{133}d_3 d_2 + \kappa_{132}d_3^2,$$

$$p_{11} = \kappa_{333}d_1^2 - 2\kappa_{133}d_1 d_3 + \kappa_{113}d_3^2,$$

$$s_2 = \tfrac{1}{2}(-\kappa_{333}d_2 + \kappa_{332}d_3),$$

$$s_1 = \tfrac{1}{2}(-\kappa_{333}d_1 + \kappa_{133}d_3),$$

$$X_{2222} = \kappa_{3333}d_2^4 - 4\kappa_{3332}d_3 d_2^3 + 6\kappa_{3322}d_3^2 d_2^2 - 4\kappa_{3222}d_3^3 d_2 + \kappa_{2222}d_3^4,$$

$$X_{1222} = \kappa_{3333}d_1 d_2^3 - \kappa_{1333}d_3 d_2^3 - 3\kappa_{3332}d_1 d_3 d_2^2 + 3\kappa_{1332}d_3^2 d_2^2 + \\ +3\kappa_{3322}d_1 d_3^2 d_2 - 3\kappa_{1322}d_3^3 d_2 - \kappa_{3222}d_1 d_3^3 + \kappa_{1222}d_3^4,$$

$$X_{1122} = \kappa_{3333}d_1^2 d_2^2 - 2\kappa_{3332}d_1^2 d_3 d_2 - 2\kappa_{1333}d_1 d_3 d_2^2 + \kappa_{3322}d_1^2 d_3^2 + \\ +4\kappa_{1332}d_1 d_3^2 d_2 + \kappa_{1133}d_3^2 d_2^2 - 2\kappa_{1322}d_1 d_3^3 - 2\kappa_{1132}d_3^3 d_2 + \kappa_{1122}d_3^4,$$

$$X_{1112} = \kappa_{3333}d_1^3 d_2 - \kappa_{3332}d_1^3 d_3 - 3\kappa_{1333}d_1^2 d_3 d_2 + 3\kappa_{1332}d_1^2 d_3^2 + \\ +3\kappa_{1133}d_1 d_3^2 d_2 - 3\kappa_{1132}d_1 d_3^3 - \kappa_{1113}d_3^3 d_2 + \kappa_{1112}d_3^4,$$

$$X_{1111} = \kappa_{3333}d_1^4 - 4\kappa_{1333}d_1^3 d_3 + 6\kappa_{1133}d_1^2 d_3^2 - 4\kappa_{1113}d_1 d_3^3 + \kappa_{1111}d_3^4,$$

$$Y_{222} = -\kappa_{3333}d_2^3 + 3\kappa_{3332}d_3 d_2^2 - 3\kappa_{3322}d_3^2 d_2 + \kappa_{3222}d_3^3,$$

$$Y_{122} = -\kappa_{3333}d_1d_2^2 + 2\kappa_{3332}d_1d_3d_2 + \kappa_{1333}d_3d_2^2 - \\ -\kappa_{3322}d_1d_3^2 - 2\kappa_{1332}d_3^2d_2 + \kappa_{1322}d_3^3,$$

$$Y_{112} = -\kappa_{3333}d_1^2d_2 + 2\kappa_{1333}d_1d_3d_2 + \kappa_{3332}d_1^2d_3 - \\ -2\kappa_{1332}d_1d_3^2 - \kappa_{1133}d_2d_3^2 + \kappa_{1132}d_3^3,$$

$$Y_{111} = -\kappa_{3333}d_1^3 + 3\kappa_{1333}d_1^2d_3 - 3\kappa_{1133}d_1d_3^2 + \kappa_{1113}d_3^3,$$

$$Z_{22} = \kappa_{3333}d_2^2 - 2\kappa_{3332}d_3d_2 + \kappa_{3322}d_3^2,$$

$$Z_{12} = \kappa_{3333}d_1d_2 - \kappa_{3332}d_1d_3 - \kappa_{1333}d_3d_2 + \kappa_{1332}d_3^2,$$

$$Z_{11} = \kappa_{3333}d_1^2 - 2\kappa_{1333}d_1d_3 + \kappa_{1133}d_3^2,$$

$$T_2 = -\kappa_{3333}d_2 + \kappa_{3332}d_3,$$

$$T_1 = -\kappa_{3333}d_1 + \kappa_{1333}d_3.$$

They satisfy the following relationships:

$$o_{\alpha\beta\gamma} = 0;\ \alpha,\beta,\gamma \in \{1,2\},$$

$$D_3^2 = \frac{3p_{11}^2}{X_{1111}} = \frac{3p_{22}^2}{X_{2222}} = \frac{3p_{11}p_{12}}{X_{1112}} = \frac{3p_{12}p_{22}}{X_{1222}} =$$

$$= \frac{p_{11}p_{22}+2p_{12}^2}{X_{1122}} == \frac{3p_{11}s_1}{Y_{111}} = \frac{3p_{22}s_2}{Y_{222}} =$$

$$= \frac{p_{11}s_2+2p_{12}s_1}{Y_{112}} = \frac{p_{22}s_1+2p_{12}s_2}{Y_{122}},$$

$$\frac{C_{33}}{D_3} = \frac{1}{p_{11}}\left(Z_{11} - \frac{2s_1^2}{D_3^2}\right) = \frac{1}{p_{12}}\left(Z_{12} - \frac{2s_1s_2}{D_3^2}\right) =$$

$$= \frac{1}{p_{22}}\left(Z_{22} - \frac{2s_2^2}{D_3^2}\right) = \frac{T_1}{3s_1} = \frac{T_2}{3s_2},$$

$$\kappa_{333} = 3C_{33}D_3 + 2qD_3^3,$$

$$-2(q^2+1) = \frac{\kappa_{1111}-3C_{11}^2}{D_1^4} = \frac{\kappa_{2222}-3C_{22}^2}{D_2^4} =$$

$$= \frac{\kappa_{3333}-3C_{33}^2}{D_3^4} = \frac{\kappa_{1122}-C_{11}C_{22}-2C_{12}^2}{D_1^2D_2^2} =$$

$$= \frac{\kappa_{1133} - C_{11}C_{33} - 2C_{13}^2}{D_1^2 D_3^2} = \frac{\kappa_{2233} - C_{22}C_{33} - 2C_{23}^2}{D_2^2 D_3^2}.$$

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VELOCITY FIELD IN A STATIONARY POINT-AXIAL GALACTIC MODEL

J. Sanz-Subirana[1], J.M. Juan-Zornoza[1], R. Cubarsi[1], S. Alcobe[2]

[1] *Universitat Politècnica de Catalunya, Barcelona, Spain*
[2] *Universitat de Barcelona, Spain*

1. Introduction

Until now, the axial symmetry has been regarded as a good approximation for stellar system models that verifies: a) the collissionless Boltzman equation, and b) the ellipsoidal hypothesis for the peculiar velocity distribution of the stars. At present however the available observations indicate that the aforementioned hypothesis is not sufficient. They are several evidences in this sense:

1) the velocity curves show a different behaviour by considering different directions in the galaxy;