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**STRUCTURE AND EVOLUTION  
OF STELLAR SYSTEMS**

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## STATIONARY AND NON-STATIONARY POTENTIALS IN A POINT-AXIAL STELLAR SYSTEM MODELS

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### 1. Introduction

We consider a stellar system model that verifies the Boltzman equation collisionless and the ellipsoidal hypothesis for the distribution of peculiar velocities of the stars. In the axial case, the potential has been studied for several authors: *Chandrasekhar (1942)*, *Català (1972)*, *Orús (1977)* and *Sala (1987)*, obtaining stationary potentials of stäkel type and time depending potentials that have three isolating integrals in involution. In this work, we determine completely the potential for a such model in the stationary and non-stationary cases.

## 2. The galactic model

We adopt a galactic model based on the collisionless Boltzmann equation:

$$\frac{d\Psi}{dt} = \frac{\partial\Psi}{\partial t} + \mathbf{V} \cdot \nabla_r \Psi - \nabla_r U \cdot \nabla_v \Psi = 0 \quad (1)$$

and the ellipsoidal hypothesis for the distribution of peculiar velocities of stars

$$\Psi(\mathbf{r}, \mathbf{v}, t) \equiv \Psi(\mathbf{Q} + \sigma) \quad (2)$$

where:  $\mathbf{Q} = \mathbf{v}^t \cdot \mathbf{A} \cdot \mathbf{v}$  and  $\mathbf{A}$  and  $\sigma$  are functions of position and time.

Equation (1) under hypothesis (2) gives the well known Fundamental equations of stellar dynamics (*Chandrasekhar, 1942*), which leads to the elements of velocity ellipsoid. A more detailed description of the model is presented in (*Sanz et al., 1989*). The stellar density  $N$ , we can express as

$$N = \frac{4\pi \int_0^\infty \Psi(s^2 + \sigma) \cdot s^2 ds}{\sqrt{|\mathbf{A}|}}$$

## 3. Results

From the study of the integrability conditions of the Fundamental equations of stellar dynamics (*Chandrasekhar, 1942*), we obtain a set of relationships between the parameters of the model which give rise to some different cases that have provided us the following solutions for the potential:

### • Stationary case:

$$U_1 = C_1(\varpi^2 + z^2) + \frac{C_2}{z^2} + C_3; \quad C_1, C_2, C_3 = \text{const},$$

$$U_2 = \frac{C_1}{\varpi^2 + z^2} + \frac{C_2}{z^2} + F(\varpi^2 + z^2); \quad C_1, C_2 = \text{const},$$

$F$  is an arbitrary function.

• Non-stationary case:

$$U_1 = \frac{1}{2k_3^2} \left( D_1 + \frac{k_3^2 - 2k_3 \ddot{k}_3}{4} \right) (\omega^2 + z^2) + \frac{C_2}{z^2} + C_3;$$

$$D_1, C_2, C_3 = \text{const},$$

$$U_2 = \frac{k_3^2 - 2k_3 \ddot{k}_3}{8k_3^2} (\omega^2 + z^2) + \frac{1}{2k_3} f \left( \frac{\omega^2 + z^2}{k_3} \right),$$

$k_3$  is a function of time,  $f$  is an arbitrary function.

Stellar density (non-stationary case, see the stationary case in (*Juan-Zornoza, Sanz-Subirana, 1991*):

$$\sigma_1 = \frac{D_1}{k_3^3} (k_3 z^2 + a(\theta) \omega^2) - \frac{\beta^2}{k_2} \frac{\omega^2}{\omega_M^2 (1 + \frac{z^2}{B}) + \omega^2},$$

where  $\omega_M^2$  corresponds to the maximum of rotation velocity;

$$\sigma_2 = g \left( \frac{\omega^2 + z^2}{k_3} \right) - \frac{\beta^2 \left( \frac{\omega^2 + z^2}{k_3} - \frac{z^2}{k_3 + b(\theta) \omega^2} \right)}{h \left( \frac{\omega^2 + z^2}{k_3}, \frac{z^2}{k_3 + b(\theta) \omega^2} \right)},$$

where  $g$  and  $h$  are arbitrary functions.

The obtained results complement of the cylindrical case obtained by *Orús (1977)* and *Sala (1986)*. As in the cylindrical case, a spherical term which corresponds to the potential created in the inner of an homogeneous sphere appears, accounting for the contribution of the galactic halo. In the other case, an arbitrary function of the distance to galactic centre must be considered.

We must point out that the obtained potentials do not depend on the  $\theta$  angle. Nevertheless the function  $\sigma$  and the stellar density  $N$  are functions of  $\theta$  (the self-consistency hypothesis is not included). Thus, our model is consistent with point-axial mass distributions.

## References

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### DISCUSSION OF HYPOTHESES AND DYNAMIC MODELS FOR STELLAR SYSTEMS

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## 1. Galactic model

### 1.1. Basic assumptions.

A) The Galactic system is interpreted as a coexistence of several stellar components, like inner and main bulge, thin and thick disk, stellar halo, corona, etc. It is not clear if these stellar components are composed of one or more stellar populations (King, 1993), but in order to benefit the mutual understanding we can refine the classification criterion up to identify components and populations. Then, we can name component to a stellar group with similar astrophysical properties and, in particular, from a kinematical viewpoint, to a group that has reached a kind of statistical equilibrium so that their velocity distribution can be assumed of normal type. Then using Chandrasekhar's notation: